Published by Institute of Physics Publishing for SISSA/ISAS
Received: *March 21, 2002*
Accepted: *June 04, 2002*

Strings on orbifolded PP-waves

Tadashi Takayanagi

Department of Physics, Faculty of Science, University of Tokyo, Hongo 7-3-1
Bunkyo-ku, Tokyo, 113-0033, Japan
E-mail: takayana@hep-th.phys.s.u-tokyo.ac.jp

Seiji Terashima

Institute for Theoretical Physics, University of Amsterdam, Valckenierstraat 65
1018 XE, Amsterdam, The Netherlands
E-mail: sterashi@science.uva.nl

Abstract: We show that the string spectrum in the pp-wave limit of $AdS_5 \times S^5/Z_M$ (orbifolded pp-wave) is reproduced from the $\mathcal{N} = 2$ quiver gauge theory by quantizing the Green-Schwarz string theory on the orbifolded pp-wave in light cone gauge. We find that the twisted boundary condition on the world-sheet is naturally interpreted from the viewpoint of the quiver gauge theory. The correction of order $g^2_{\rm YM}$ to the gauge theory operators agrees with the result in its dual string theory. We also discuss strings on some other orbifolded pp-waves.

Keywords: 1/N Expansion, D-branes, AdS/CFT and dS/CFT Correspondence, Extended Supersymmetry.

JHEP06(2002)036

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1. Introduction

The AdS/CFT correspondence [1, 2] gives us a deep understanding of duality between string theory and gauge theory. Furthermore, from this we would know how the closed string theory can be realized in terms of non-gravitational theory. In the most interesting case $AdS_5 \times S^5$ the conjectured duality to $\mathcal{N}=4$ super Yang-Mills theory has been checked on the supergravity level [3, 4]. In order to show this in full string theory we must analyze the world-sheet theory in the presence of RR-flux. Recently, an intriguing progress has been made in this direction [5]. In the papers [6, 7] it was shown that the Penrose's limit [8] of $AdS_5 \times S^5$ in type-IIB string theory is exactly solvable (see also related papers [9, 10, 11]). The background in this limit is described by the maximally supersymmetric pp-wave (plane-fronted wave with parallel rays) with RR-flux [12, 13].

The explicit metric of pp-wave [14] is given by

$$ds^2 = -4dx^+dx^- - \mu^2 \sum_{i=1}^{8} \left(z^i\right)^2 \left(dx^+\right)^2 + \sum_{i=1}^{8} \left(dz^i\right)^2 , \tag{1.1}$$

where we have defined the light-cone coordinate $x^\pm = (x^0 \pm x^9)/2$ and the RR-flux $F_{+1234} = F_{+5678}$ is proportional to μ. Using this solvable limit the authors of [5] construct the explicit map between states of the string theory in pp-wave background and the operators in $\mathcal{N}=4$ super Yang-Mills theory which have large U(1) R-charge $J \sim N^{1/2}$. These operators can be regarded as "almost chiral primary" operators and the deviation from chiral primary represents stringy excitations.

In this paper we would like to consider the pp-wave limit of orbifolded $AdS_5 \times S^5$, especially $AdS_5 \times S^5/Z_M$ (see [15]–[18] for the tests of duality on the supergravity level). The orbifold action acts on the four coordinates (z_6, z_7, z_8, z_9) in the background (1.1). This

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example is interesting because of two reasons. First we can discuss the less supersymmetric $\mathcal{N}=2$ quiver gauge theory [15, 16, 19, 20] (see also papers [21, 22, 23] for the enhancement of $\mathcal{N}=4$ supersymmetry in the pp-wave limit of $\mathcal{N}=1$ gauge theory). Secondly an orbifold is a good example of a stringy geometry and thus it will be important to see how its structure can be seen in the gauge theory side. The duality in this orbifolded pp-wave was already implied in [5] and the particular case $M=2$ was briefly discussed in [21]. Below we investigate the detailed correspondence for the Z_M orbifolded string theory in both the untwisted sector and twisted sectors. In particular we explicitly quantize the world-sheet theory on the orbifolded pp-waves. This analysis reveals the interesting interpretation of the twisted boundary condition in the orbifolded world-sheet theory from the viewpoint of the $\mathcal{N}=2$ quiver gauge theory. This enables us to see that the string spectrum is exactly matched with the gauge theory calculation up to order $g(\sim g_{\rm YM}^2)$ correction. We also mention a similar analysis for more general orbifolds.

The contents of this paper is as follows. In section 2 we briefly review the results in [5]. In section 3 we quantize the orbifolded string theory on pp-wave and discuss its correspondence with the $\mathcal{N}=2$ quiver gauge theory. In section 4 we mention general orbifolds. In section 5 we draw conclusions.

While preparing this paper for publication, we received the preprints [24, 25] which have some overlap with ours.

2. String on pp-waves and $\mathcal{N}=4$ super Yang-Mills

In [5] it was proposed that the AdS/CFT correspondence [1] in the Penrose's limit is the duality between the string theory in the pp-wave background (1.1) and the $\mathcal{N}=4$ super Yang-Mills theory. The near horizon limit of a system of N D3-branes is described by the geometry $AdS_5\times S^5$ and its explicit metric is given by

$$ds^2 = R^2\left(-dt^2\cosh^2\rho + d\rho^2 + \sinh^2\rho\, d\Omega_3^2 + d\psi^2\cos^2\theta + d\theta^2 + \sin^2\theta d\Omega_3'^2\right), \qquad (2.1)$$

where $R=(4\pi g N\alpha'^2)^{1/4}$ is the radius of S^5. For this background the Penrose's limit, which is generally defined as the limiting procedure of taking infinitely small neighborhoods of null geodesic, is given by the limit $R\to\infty$ focusing on $\rho=\theta=0$ with the following scaling

$$x^+ = \frac{t+\psi}{2}, \qquad x^- = \frac{R^2(t-\psi)}{2}, \qquad \rho = \frac{r}{R}, \qquad \theta = \frac{y}{R}. \qquad (2.2)$$

Then we obtain exactly the pp-wave background (1.1) with $\mu=1$.

The pp-wave background (1.1) in light-cone Green-Schwarz(GS) string theory is described by the following world-sheet lagrangian [6, 7]

$$\mathcal{L} = \frac{1}{2}\left[\partial_+ z^i\partial_- z^i - (2\pi\alpha' p^+\mu)^2(z^i)^2\right], \qquad (2.3)$$

where z^i $(i=1,2,\ldots,8)$ are the scalar fields 3 corresponding to the spacetime coordinates not in the light-cone direction and we omit the terms which include sixteen fermions S^{1a} and S^{2a} $(a = 1, 2, \ldots, 8)$. In this paper we mainly show the analysis of bosonic operators because the fermionic sector can be examined similarly.

An operator of conformal dimension Δ and R-charge J in the gauge theory side corresponds to a state in the Green-Schwarz string which has the light-cone momentum p^-, p^+ by the following rule

$$2p^- = \Delta - J, \qquad 2p^+ = \frac{\Delta + J}{R^2}\,. \tag{2.4}$$

In order to keep the value of p^+ and p^- finite we focus on the operator $\Delta \sim J \sim N^{\frac{1}{2}}$ in the large-N limit. We always assume this scaling below in this paper.

In this duality [5] the ground state $|\mathrm{vac}, p^- = 0, p^+\rangle$ of string theory in pp-wave background is identified with the operator $\mathrm{tr}[Z^J]$, which has the conformal dimension $\Delta = J$. The complex scalar field Z represents the transverse complex scalar field whose spin corresponds to the value J. The excitations in the string theory correspond to the insertions of fields Φ^i (4 transverse scalars), $\chi^a_{J=1/2}$ (8 fermions) or covariant derivatives D_i $(i = 1, 2, 3, 4)$ in $\mathrm{tr}[Z^J]$. The map between them is roughly given as follows

$$a_n^{\dagger i} \longrightarrow D_i Z \quad (i = 1, 2, 3, 4)\,, \tag{2.5}$$

$$a_n^{\dagger j} \longrightarrow \Phi^j \quad (j = 5, 6, 7, 8)\,, \tag{2.6}$$

$$S_n^{\dagger a} \longrightarrow \chi^a_{J=1/2} \quad (a = 1, 2, \ldots, 8)\,, \tag{2.7}$$

where $a_n^{\dagger i}$ and $S_n^{\dagger a}$ denote the bosonic and fermionic oscillators with the level n (following the convention in [5]) of the Green-Schwarz string in the light-cone gauge.

The detailed map including the level n is more complicated and shows the way how the stringy excitations are translated into the gauge theoretic counterparts [5]. Let us consider the insertion of Φ^{i_j} operators s times into the $\mathrm{tr}[Z^J]$ at just before l_j-th Z and summing over l_j

$$\begin{aligned}
&\sum_{l_i=0}^{J-1}\prod_{i=1}^{s}\exp\left(2\pi i\frac{l_i n_i}{J}\right)\mathrm{tr}\left((Z)^{k_1}\Phi(Z)^{k_2}\Phi\cdots(Z)^{k_s}\Phi(Z)^{J-\sum_{i=1}^{s}k_s}\right) = \\
&\quad = \sum_{l_i=0}^{J-1}\exp\left(2\pi i\frac{l_1\sum_{j=1}^{s}n_j}{J}\right)\times \\
&\quad\times\prod_{i=1}^{s}\exp\left(2\pi i\frac{(l_i-l_1)n_i}{J}\right)\mathrm{tr}\left((Z)^{k_1}\Phi\cdots(Z)^{k_s}\Phi(Z)^{J-\sum_{i=1}^{s}k_s}\right),
\end{aligned} \tag{2.8}$$

where the value $\sum_{j=1}^{i} k_j$ is equal to the i-th smallest value among l_j's. Then it vanishes except $\sum_{i=1}^{s} n_i = 0$ by summing up l_1 with fixing $l_i - l_1$ using the cyclic property of the trace. Thus we must require this relation. Even though there is an ambiguity on the correct ordering at $l_i = l_j$, it will not be important in the "dilute gas" approximation [5]. Note that if one regards the positions of Φ^{i_j} as the physical positions of s point particles in 1+1 dimensional spacetime Σ, the discrete Fourier transformation in (2.8) defines the "momentum" n_i for each particle and the relation $\sum_{i=1}^{s} n_i = 0$ is interpreted as momentum conservation. Furthermore, if we identify Σ with the string would-sheet, then the "string of Z's" can be regarded as the physical string as argued in [5].

In string theory side we can interpret the operator (2.8) as

$$\prod_{j=1}^{s} \left(\alpha_{n_j}^{\dagger i_j} \right) |\mathrm{vac}, 0, p^+\rangle \, . \tag{2.9}$$

Note that the above state satisfies the level matching condition $\sum_i n_i = 0$ as expected. In string theory side we can compute the light-cone energy

$$2p^- = \sum_{j=1}^{s} \sqrt{1 + \frac{n_i^2}{(\alpha' p^+)^2}} \, . \tag{2.10}$$

The duality tells us the relation (2.4) and we obtain

$$\Delta - J = \sum_{j=1}^{s} \sqrt{1 + \frac{4\pi g N n_i^2}{J^2}} \, . \tag{2.11}$$

This prediction can be checked in the first order of the coupling constant g by the calculation in the large-N limit of $\mathcal{N} = 4$ super Yang-Mills theory [5].

3. Strings on Z_M orbifolded pp-wave and $\mathcal{N} = 2$ quiver gauge theory

The low energy limit of N D3-branes on the orbifold C^2/Z_M can be described by the $\mathcal{N} = 2$ quiver gauge theory [19], where the orbifold action is defined as follows

$$(\phi_1, \phi_2) \longrightarrow \left(e^{2\pi i/M} \, \phi_1, e^{-2\pi i/M} \, \phi_2 \right) \, , \tag{3.1}$$

where $\phi_1 = z_6 + iz_7$ and $\phi_2 = z_8 + iz_9$ denote the coordinates of C^2/Z_M (see (1.1)). In the near horizon limit we obtain the geometry $AdS_5 \times S^5/Z_M$ and AdS/CFT correspondence tells us the duality between this background and the $\mathcal{N} = 2$ gauge theory [15]–[18]. The metric of this background is given by the metric (2.1) orbifolded by the Z_M action only on Ω_3' and the value of radius is given by $R = (4\pi g N M \alpha'^2)^{\frac{1}{4}}$ because we start with MN D3-branes and further we impose the Z_M projection. In this section we would like to investigate the AdS/CFT correspondence by taking the Penrose's limit.

3.1 GS string on orbifolded pp-waves

The Penrose's limit of $AdS_5 \times S^5/Z_M$ is the Z_M orbifold of the pp-wave background (1.1). Since the orbifold projection does not act on the light-cone direction, we can describe GS string theory in this background simply by taking the Z_M orbifold in the sense of world-sheet theory in light-cone gauge (refer to [6, 7] for the detailed analysis of GS string theory on pp-wave). Then the orbifold action in string theory is defined by (3.1) identifying the coordinate z^i and ϕ_i with the world-sheet scalar fields.

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The m-th twisted sector is defined by the following boundary condition

$$
\begin{aligned}
z^i(\sigma+1,\tau) &= z^i(\sigma,\tau)\,, \quad (i=1,2,3,4)\\
\phi_1(\sigma+1,\tau) &= e^{2\pi i m/M}\,\phi_1(\sigma,\tau)\,,\\
\phi_2(\sigma+1,\tau) &= e^{-2\pi i m/M}\,\phi_2(\sigma,\tau)\,.
\end{aligned} \tag{3.2}
$$

We have also sixteen real fermions S^{1a} and S^{2a} and they are also divided into two groups. The eight fermions in one group obey the trivial boundary condition and the other eight are twisted by the phase $e^{\pm 2\pi i/M}$.

For example, the mode expansions of (twisted) bosons ϕ_1 are given by

$$
\begin{aligned}
\phi_1(\sigma,\tau) &= i\sum_{n\in\mathbf{Z}}\left(\frac{1}{\omega_n}e^{2\pi i(n-\delta)\sigma-i\omega_n\tau}\,\alpha^{1L}_{n-\delta}+\frac{1}{\omega'_n}e^{-2\pi i(n+\delta)\sigma-i\omega'_n\tau}\,\alpha^{1R}_{n+\delta}\right),\\
\bar\phi_1(\sigma,\tau) &= i\sum_{n\in\mathbf{Z}}\left(\frac{1}{\omega'_n}e^{2\pi i(n+\delta)\sigma-i\omega'_n\tau}\,\bar\alpha^{1L}_{n+\delta}+\frac{1}{\omega_n}e^{-2\pi i(n-\delta)\sigma-i\omega_n\tau}\,\bar\alpha^{1R}_{n-\delta}\right),
\end{aligned} \tag{3.3}
$$

where $\delta = m/M$ and we can obtain the expression of ϕ_2 by replacing δ with $-\delta$. Here we also defined

$$
\begin{aligned}
\omega_n &= \pm 2\pi\sqrt{(n-\delta)^2+(\alpha' p^+\mu)^2} \quad (+\text{ for } n>0,-\text{ for } n\le 0)\,,\\
\omega'_n &= \pm 2\pi\sqrt{(n+\delta)^2+(\alpha' p^+\mu)^2} \quad (+\text{ for } n\ge 0,-\text{ for } n< 0)\,.
\end{aligned} \tag{3.4}
$$

Then the usual canonical quantization of the lagrangian (2.3) gives

$$
\begin{aligned}
\left[\alpha^{1L}_{n-\delta},\bar\alpha^{1L}_{n'+\delta}\right] &= \omega_n\delta_{n+n',0}\,, \qquad \left[\alpha^{1R}_{n+\delta},\bar\alpha^{1R}_{n'-\delta}\right] = \omega'_n\delta_{n+n',0}\,,\\
\left[\alpha^{2L}_{n+\delta},\bar\alpha^{2L}_{n'-\delta}\right] &= \omega'_n\delta_{n+n',0}\,, \qquad \left[\alpha^{2R}_{n-\delta},\bar\alpha^{2R}_{n'+\delta}\right] = \omega_n\delta_{n+n',0}\,.
\end{aligned} \tag{3.5}
$$

The vacuum state of the world-sheet theory in the m-th twisted sector is defined by

$$
\begin{aligned}
\alpha^{1L}_{n-\delta}|\text{vac}\rangle_m = \bar\alpha^{1R}_{n-\delta}|\text{vac}\rangle_m = \alpha^{2R}_{n-\delta}|\text{vac}\rangle_m = \bar\alpha^{2L}_{n-\delta}|\text{vac}\rangle_m = 0 \quad (n\ge 1)\,,\\
\alpha^{1R}_{n+\delta}|\text{vac}\rangle_m = \bar\alpha^{1L}_{n+\delta}|\text{vac}\rangle_m = \alpha^{2L}_{n+\delta}|\text{vac}\rangle_m = \bar\alpha^{2R}_{n+\delta}|\text{vac}\rangle_m = 0 \quad (n\ge 0)\,.
\end{aligned} \tag{3.6}
$$

The bosonic part of the light-cone hamiltonian is given by

$$
\begin{aligned}
H = {} & H_0 + H_{\text{flat}} + \frac{1}{2\pi\alpha' p^+}\sum_{n=0}^{\infty}\left[\alpha^{1L}_{-n-\delta}\bar\alpha^{1L}_{n+\delta}+\bar\alpha^{1R}_{-n-\delta}\alpha^{1R}_{n+\delta}\right]+\\
& + \frac{1}{2\pi\alpha' p^+}\sum_{n=1}^{\infty}\left[\bar\alpha^{1L}_{-n+\delta}\alpha^{1L}_{n-\delta}+\alpha^{1R}_{-n+\delta}\bar\alpha^{1R}_{n-\delta}\right]+(\phi_2\text{ part})\,,
\end{aligned} \tag{3.7}
$$

where H_0 and H_{flat} denote the contributions from zero-modes and four bosons in the non-orbifold direction (x^1,x^2,x^3,x^4). We omit these detailed forms because both contributions are the same as in [7].

For example, the oscillators $\alpha^{1L}_{-n-\delta}$ and $\bar\alpha^{1R}_{-n-\delta}$ in the m-th twisted sector have the light-cone energy

$$
2p^- = \sqrt{\mu^2+\frac{(n+\delta)^2}{(\alpha' p^+)^2}}\,. \tag{3.8}
$$

We assume that the zero-energy (or the value of p^- for the lowest state) in the twisted sector is zero and will see that the desirable spectrum is obtained in twisted sectors later. Note also that if we take the limit $\mu \to 0$, then we reproduce the mass spectrum of familiar string theory on the orbifold C^2/Z_M.

Then the spectrum should be Z_M projected and also satisfy the level matching condition. The level is defined to be the summation of "$n\pm\delta$" with respect to each oscillator for a given state. The level in the left-moving sector should be equal to that in the right-moving sector.

Now it is convenient to extract the creation operators and we rename them as follows

$$\begin{aligned} \alpha^{\dagger 1}_n &\equiv \alpha^{1L}_{n-\delta} \quad (n \le 0)\,, \qquad \alpha^{1R}_{-n+\delta} \quad (n > 0)\,, \\ \bar{\alpha}^{\dagger 1}{}_n &\equiv \bar{\alpha}^{1L}_{n+\delta} \quad (n < 0)\,, \qquad \bar{\alpha}^{1R}_{-n-\delta} \quad (n \ge 0)\,, \\ \alpha^{\dagger 2}_n &\equiv \alpha^{2L}_{n+\delta} \quad (n < 0)\,, \qquad \alpha^{2R}_{-n-\delta} \quad (n \ge 0)\,, \\ \bar{\alpha}^{\dagger 2}{}_n &\equiv \bar{\alpha}^{2L}_{n-\delta} \quad (n \le 0)\,, \qquad \bar{\alpha}^{2R}_{-n+\delta} \quad (n > 0)\,. \end{aligned} \tag{3.9}$$

Note also the Z_M action

$$\begin{aligned} &\alpha^{\dagger 1}_n \longrightarrow e^{2\pi i\delta}\alpha^{\dagger 1}_n\,, \qquad \bar{\alpha}^{\dagger 1}{}_n \longrightarrow e^{-2\pi i\delta}\bar{\alpha}^{\dagger 1}{}_n\,, \\ &\alpha^{\dagger 2}_n \longrightarrow e^{-2\pi i\delta}\alpha^{\dagger 2}_n\,, \qquad \bar{\alpha}^{\dagger 2}{}_n \longrightarrow e^{2\pi i\delta}\bar{\alpha}^{\dagger 2}{}_n\,, \end{aligned} \tag{3.10}$$

and the level of left-mover minus that of right-mover for $\alpha^{\dagger 1}_n$, $\bar{\alpha}^{\dagger 1}{}_n$, $\alpha^{\dagger 2}_n$ and $\bar{\alpha}^{\dagger 2}{}_n$ are given by $n-\delta$, $n+\delta$, $n+\delta$ and $n-\delta$, respectively. Then we can construct all states by operating these on the m-th twisted vacuum and requiring the level matching condition and invariance under the Z_M action. Note that there are two types of Z_M invariants, i.e. a product of M oscillators in (3.9) which have the same "Z_M charge" of (3.10) and products of different "Z_M charge" oscillators. The differences of the levels of these invariants are integer.

Finally let us briefly mention the fermionic zeromodes. The orbifold projection acts on the fermionic fields as they are space-time spinors. In untwisted sector the fermion zero modes S^{1a}_0, S^{2b}_0, $(a, b = 1, 2, \ldots, 8)$ satisfy the relation $\{S^{ia}_0, S^{jb}_0\} = \delta_{ab}\delta_{ij}$. We can quantize these and get 2^8 "massless" states if we consider the flat space not the orbifold. In the orbifold theory the Z_M acts on them as phase factor $e^{2\pi i(s_3-s_4)/M}$ where s_3 and s_4 are spin along (x_6, x_7) and (x_8, x_9), respectively, and $s_3, s_4 = \pm 1/2$. In the twisted sector we have eight fermion zero modes S^a_0 and after the quantization we have $2^4 = 8\,\text{bosons} + 8\,\text{fermions}$ "massless" states. Four bosons parameterize the hyperkahler moduli of the vanishing cycles and the other four bosons explain the RR field moduli if we set $\mu = 0$.

3.2 $\mathcal{N} = 2$ quiver gauge theory and string spectrum

The quiver gauge theory is defined by the corresponding quiver diagram [19, 20]. In our case we consider the quiver diagram of A_{M-1} type, which has M nodes and arrows between them. Each node represents a gauge group $\mathrm{U}(N)$ and we denote the corresponding i-th vector multiplet as (Z_i, W_i) in the $\mathcal{N} = 1$ superfield notation. Thus its gauge group is $G = \prod_{i=1}^{M} \mathrm{U}(N)$. It should be also noted that the gauge coupling g' of each $\mathrm{U}(N)$ gauge theory is given by $g' = Mg$ [15, 16].

Field	$J=r/2$	$2j$	gauge charge
Z	1	0	adj
Q^1	0	1	$(\bar{N},N)$
Q^2	0	1	$(N,\bar{N})$
$\bar{Q}^1$	0	-1	$(N,\bar{N})$
$\bar{Q}^2$	0	-1	$(\bar{N},N)$

Table 1: R-charge.

In the quiver diagram the arrow pointing one direction corresponds to the superfield $Q_i^1, \bar{Q}_i^2$, which is bifundamental matters $(\bar{N},N)$ with respect to i-th and $i+1$-th gauge groups, and the other arrow to Q_i^2, $\bar{Q}_i^1$. These form M hyper multiplets $(Q_i^1, \bar{Q}_i^2)$, $(i=1,\ldots,M)$. Below we regard the index i as Z_M valued. The R-charges of these fields are summarized in table 1.

Here j and r are the spin of $\mathrm{SU}(2)_R$ and $\mathrm{U}(1)_R$, respectively. This gauge theory has the $\mathcal{N}=2$ superconformal symmetry. The chiral primary operators are operators with $\Delta = 2j + r/2$ [26]. Below we also use the $NM \times NM$ matrices Z, Q^1, Q^2 before the Z_M projection and they are defined as follows

$$Z = \begin{pmatrix} Z_1 & 0 & \cdots & \cdots & 0 \\ 0 & Z_2 & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & Z_M \end{pmatrix}, \qquad Q = \begin{pmatrix} 0 & Q_1 & 0 & \cdots & 0 \\ 0 & 0 & Q_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & & \ddots & Q_{M-1} \\ Q_M & 0 & \cdots & \cdots & 0 \end{pmatrix}. \tag{3.11}$$

After we take the pp-wave limit, we have again the translation rule (2.4) and we should consider the operators which have $\Delta \sim J \sim \sqrt{N}$ only. From the table 1, we see that they are operators constructed from J Z's and finite, but, arbitrary numbers of Q's and $\bar{Q}$'s.[1] Then we can naturally identify the translation rule of the gauge theory operators into the operators in the string theory:

$$\left(\alpha_n^{\dagger 1}, \bar{\alpha^{\dagger 1}}_n, \alpha_n^{\dagger 2}, \bar{\alpha^{\dagger 2}}_n\right) \longleftrightarrow \left(Q^1, \bar{Q^1}, Q^2, \bar{Q^2}\right). \tag{3.12}$$

The translation rule with respect to the other bosonic fields in the gauge theory is the same as in the $\mathcal{N}=4$ case (2.5).

The light-cone energy (3.8) is rewritten in the gauge theoretic language as follows

$$\Delta - J = \sqrt{1 + \frac{4\pi g N M (n+\delta)^2}{J^2}} = \sqrt{1 + \frac{4\pi g' N (n+\delta)^2}{J^2}}, \tag{3.13}$$

Notice that t' Hooft coupling again appears correctly.

Next let us consider the correspondence between string states and field theory operators in detail. In the string theory, there are an untwisted and $M-1$ twisted sectors. To identify the corresponding 3 operators we define the symbol P, which acts on Z, Q and $\bar{Q}$ such that

[1] In this paper, we only consider the single trace operators. The operators of $\Delta - J = 2, 3, \ldots$, such as $\bar{Z}, \partial Q, \ldots$ also are not considered since we expect such operators have a large conformal dimension in $N \to \infty$ limit as argued in [5].

the Z_M indices are shifted by 1. From this, we can define M projectors

$$P_m \equiv \frac{1}{M}\sum_{j=0}^{M-1}\exp\left(\frac{2\pi imj}{M}\right)P^j\,. \tag{3.14}$$

Using the projectors, we can decompose any operator constructed from Z, Q and $\bar{Q}$ as

$$\mathcal{O}_m = P_m\mathcal{O}(Z,Q,\bar{Q})\,. \tag{3.15}$$

Then $\mathcal{O}_m$ is naively expected to correspond to the m-th twisted sector.

The most simple example is the following operator

$$P_m\,\mathrm{tr}\left[(Z_1)^J\right] = \frac{1}{M}\sum_{j=0}^{M-1}\exp\left(\frac{2\pi imj}{M}\right)\mathrm{tr}\left[(Z_j)^J\right], \tag{3.16}$$

which will couple to the m-th twisted sector and correspond to the m-th twisted sector vacuum. This speculation can be proved exactly as follows. First note that the disk amplitude $\mathcal{A}$ of the m-th twisted field in closed string and the open string massless field Z is given[2] by

$$\mathcal{A} = \mathrm{Tr}\left[\gamma_m\cdot Z^J\right]\cdot\left\langle V_m(V_Z)^J\right\rangle\,, \tag{3.17}$$

where V_m and V_Z are vertex operators of the m-th twisted closed string and the open string, respectively, $\langle\cdots\rangle$ is the correlation function and the integration over moduli of world-sheet. The Chan-Paton matrix γ_m represents the "Chan-Paton factor" for m-th twisted sector as introduced in [19, 27]. Its explicit form is the following diagonal matrix.

$$\gamma_m = \mathrm{diag}\left(1, e^{2\pi im/M}, e^{4\pi im/M}, \ldots, e^{2(M-1)\pi im/M}\right). \tag{3.18}$$

Then the trace of Chan-Paton factor is given by $e^{2\pi imj/M}$ and thus we finish the proof.

These simple operators (3.16) are chiral primary and their conformal dimensions are given by $\Delta - J = 0$. Note that for these states the spin of R-symmetry $\mathrm{SU}(2)_R$ vanishes. In the string theory side we can identify $\mathcal{O}_m$ with the oscillator vacuum state $|\mathrm{vac}, p^- = 0, p^+\rangle_m$ in m-th twisted sector.

Next we would like to consider operators corresponding to the excited states. We cannot insert a single operator which has a non-zero $\mathrm{SU}(2)_R$ charge because of Z_M projection. This is consistent with the string theoretic interpretation (3.12). Note that this phenomenon is different from the $\mathcal{N}=4$ case [5], where we can insert such a single operator if the "momentum" vanishes $n=0$. Thus let us examine more than two excitations. Then a naive guess is, for example, simply following the procedures (2.8) and (3.14)

$$\sum_{j=0}^{M-1}\exp\left(\frac{2\pi imj}{M}\right)\times\sum_{k=0}^{J}\exp\left(\frac{2\pi ikn}{J}\right)\times\mathrm{tr}\left[(Z_j)^kQ'(Z_{j+1})^{J-k}Q'\right], \tag{3.19}$$

[2]Here we defined Tr as the trace for a $NM\times NM$ matrix, while tr as the trace for a $N\times N$ matrix.

where Q' represents Q^i or $\bar{Q}^i$. This is, however, incorrect because they do not reproduce the level matching condition. For instance, in $M = 2$ case the particular operator with $n = 0$

$$\sum_{j=0}^{1} \exp\left(\pi i m j\right) \times \sum_{k=0}^{J} \mathrm{tr}\left[(Z_j)^k Q_j^1 (Z_{j+1})^{J-k} Q_{j+1}^1\right]. \qquad (3.20)$$

vanishes except $m = 0$ since the $\mathrm{tr}[\cdots]$ takes the same value for $j = 0$ and $j = 1$. However, we have seen that there exists the string state $\alpha_0^{\dagger 1}\alpha_1^{\dagger 1}|\mathrm{vac}, 0, p^+\rangle$, which satisfies the level matching condition. Moreover, we can see that the corrections to $\Delta - J$ of order g_s are not reproduced by (3.19). Thus this naive definition is problematic and we should modify the identification.

Remembering the computation of previous disk amplitude (3.17) we would like to propose that the string state

$$\prod_{i=1}^{s}\left(\alpha_{n_i}^{\dagger j_i}\right)|\mathrm{vac}, 0, p^+\rangle, \qquad (3.21)$$

corresponds to the following operator instead of (3.19)

$$\begin{aligned}
&\sum_{l_i=0}^{J-1}\prod_{i=1}^{s}\exp\left(2\pi i\frac{l_i(n_i+\epsilon_i\delta)}{J}\right)\mathrm{Tr}\left(\gamma_m Z^{k_1}Q'Z^{k_2}Q'\cdots Z^{k_s}Q'Z^{J-\sum_{i=1}^{s}k_i}\right) = \\
&\quad = \sum_{l_i=0}^{J-1}\exp\left(2\pi i\frac{l_1\sum_{j=1}^{s}(n_j+\epsilon_j\delta)}{J}\right)\prod_{i=1}^{s}\exp\left(2\pi i\frac{(l_i-l_1)(n_i+\epsilon_i\delta)}{J}\right)\times \\
&\quad \times\ \mathrm{Tr}\left(\gamma_m Z^{k_1}Q'Z^{k_2}Q'\cdots Z^{k_s}Q'Z^{J-\sum_{i=1}^{s}k_i}\right), \qquad (3.22)
\end{aligned}$$

where we use the same convention as in (2.8) and ϵ_i is a sign of Q"'s fractional level.[3] The operators Q' inserted in front of the l_i-th Z are equivalent to the string state (3.21) via the rule (3.12). Note that here we used the trace Tr for $NM \times NM$ matrices and we assumed that the fields Z and Q' are Z_M projected. Then we can see that the level matching condition is satisfied because the last line of (3.22) depends only on $(l_i - l_1)$. Therefore we can find the complete correspondence between the string spectrum and operators in the gauge theory.

The shift of moding n_i by $\epsilon_i\delta$ in the above operator can be explained as follows. It should be required that the operator like (3.22) is invariant under the shift of the position Q'. In particular we can shift the value of l_i by J which corresponds to the rotation of Q' in the trace. This is because we can interpret l_i/J as the world-sheet periodic coordinate σ in (3.3) following the general idea of [5] that the "string of Z's" can be regarded as the physical string. If we move one of the fields $Q^1, \bar{Q}^2$ (or $Q^2, \bar{Q}^1$) so that it crosses the matrix γ_m, then we have the extra factor $e^{2\pi i\delta}$ (or $e^{-2\pi i\delta}$). A typical example is shown below

$$\mathrm{Tr}\left[\gamma_m Z^k \bar{Q}^1 Z^{J-k} Q^1\right] = e^{2\pi i\delta}\,\mathrm{Tr}\left[\gamma_m Z^{J-k} Q^1 Z^k \bar{Q}^1\right], \qquad (3.23)$$

[3] Here we defined the values of ϵ_i such that $\epsilon_i = 1$ for $\bar{Q}^1$, Q^2 and $\epsilon_i = -1$ for $Q^1, \bar{Q}^2$. Then $n_i + \delta\epsilon_i$ is equal to the level in (3.9).

where we have employed the commutation relation $Q^1\gamma_m = e^{2\pi i\delta}\gamma_m Q^1$. In order to cancel this extra factor we must shift the value of n_i by $\epsilon_i\delta$. Actually under the shift of l_i the summation of (3.22). In other words, the presence of the matrix γ_m is equivalent to the existence of Wilson line in the "periodic direction σ" and thus it shifts the value of momentum. In this way the quiver gauge theory operator (3.22) explicitly shows the twisted boundary condition in the string theory of the orbifolded pp-wave!

Let us examine the operator (3.22) in the explicit examples which include two hyper multiplet fields. There are four possibilities of hyper multiplets: $Q^1\bar{Q}^1$, $Q^2\bar{Q}^2$, Q^1Q^2 and $\bar{Q}^1\bar{Q}^2$. The ordering of Q's is important and is distinguished by the sign of the "momentum" (moding) $n \in \mathbf{Z}$. These states each correspond to the following four states in pp-wave string theory (see the rule (3.12))

$$\alpha^1_n\bar{\alpha}^1_{-n}|\text{vac}\rangle\,, \qquad \alpha^2_n\bar{\alpha}^2_{-n}|\text{vac}\rangle\,, \qquad \alpha^1_n\alpha^2_{-n}|\text{vac}\rangle\,, \qquad \bar{\alpha}^2_n\bar{\alpha}^1_{-n}|\text{vac}\rangle\,. \tag{3.24}$$

Note that these states satisfy the level matching condition and the Z_M invariance.

We can construct the dual operator in the gauge theory side by using the general formula (3.22). Then we obtain the following result in the Q_1Q_2 case (up to the normalization factor) after taking the trace with respect to $j = 0, 1, \ldots, M-1$

$$\sum_{j=0}^{M-1} \exp\left(2\pi i j\delta\right) \times \sum_{k=0}^{J-1} \exp\left(\frac{2\pi i k\,(n-\delta)}{J}\right) \times \text{tr}\left[(Z_j)^k Q^1_j (Z_{j+1})^{J-k} Q^2_j\right], \tag{3.25}$$

and similar results can be shown in the other three cases.

We can check that this operator has the correct conformal dimension in the large-N limit

$$\Delta - J = 2\sqrt{1 + \frac{4\pi g' N(n-\delta)^2}{J^2}} \simeq 2 + \frac{4\pi g' N(n-\delta)^2}{J^2}\,, \tag{3.26}$$

to the first order in the coupling $g' \sim g^2_{\text{YM}}$ as in the paper [5].[4] The correction proportional to $g'N/J^2$ comes from the interaction $\sim \sum_{j=0}^{M-1}(Z_jQ^1_j\bar{Z}_{j+1}\bar{Q}^1_{j+1} + Z_{j+1}\bar{Q}^1_{j+1}\bar{Z}_jQ^1_j) +$ ($Q^2Z\bar{Q}^2\bar{Z}$ term). We have also used the "dilute gas" approximation as in [5].[5]

We can also generally check that the spectrum of the string theory is consistent with the quiver gauge theory result at least up to g' order for the operator form (3.22)

$$\Delta - J = \sum_{i=1}^{s}\sqrt{1 + \frac{4\pi g' N(n_i+\epsilon_i\delta)^2}{J^2}} \simeq s + \sum_{i=1}^{s}\frac{2\pi g' N(n_i+\epsilon_i\delta)^2}{J^2}\,. \tag{3.27}$$

Finally we would like to mention the gauge theory dual of the string states which include the fermionic operators $S^{\dagger a}_n$ $(a = 1, 2, \ldots, 8)$. There are four fermionic operators which are Z_M invariant. They correspond to fermions in the vector multiplets of $\mathcal{N} = 2$ quiver gauge theory. The other four operators which are not Z_M invariant correspond to fermions in the hyper multiplets. Both of them can be treated in the same way as Z and Q', respectively, and thus we omit the detailed discussion.

[4] There is a subtle point in this argument for the operators which are not chiral primary. Here we assume the momentum independent contributions vanish if $n = m = 0$.

[5] One can see that the interaction of the form $Q\bar{Q}Q\bar{Q}$ does not affect our arguments in the dilute gas approximation. Then we expect this term can be ignored.

4. General orbifolds

Here let us briefly discuss the generalization of previous results for other two-dimensional orbifolds C^2/Γ. First consider non-abelian supersymmetric orbifolds which are classified by $A(=Z_M)$, D, E series [20, 27]. In these orbifolds each twisted sector is labeled by the conjugacy class $\mathcal{C}_\beta$, $(\beta = 1, 2, \ldots, r)$ of Γ. The quiver gauge theories correspond to the regular representation $\rho_{reg} = \oplus_{\alpha=1}^{r} n_\alpha \rho_\alpha$, where ρ_α $(1 \leq \alpha \leq r)$ are irreducible representations of Γ and we defined $n_\alpha \equiv \dim \rho_\alpha$. Their gauge groups are given by $G = \prod_{\alpha=1}^{r} \mathrm{U}(Nn_\alpha)$ and each gauge coupling is given by $\tau_\alpha = n_\alpha \tau/|\Gamma|$ $(\tau = i/g + \chi)$. In order to know the coupling of fields in the gauge theory to the $\mathcal{C}_\beta$ twisted sector we have only to replace (3.14) with

$$\frac{1}{|\Gamma|}\sum_{\alpha=1}^{r} \chi_\alpha(\mathcal{C}_\beta) P_\alpha \, , \tag{4.1}$$

where $\chi_\alpha(g) = \mathrm{tr}_\alpha(g)$ denotes the character of an element of $g \in \Gamma$ in ρ_α representation. The symbol P_α denotes the projection onto the gauge group $\mathrm{U}(Nn_\alpha)$. All other arguments (e.g. (3.22)) in the section 3 three can be generalized for D and E cases.

It is also interesting to consider non-supersymmetric orbifolds. For example, let us consider the Z_M orbifold defined by

$$\phi_1 \longrightarrow e^{2\pi i/M}\phi_1, \qquad \phi_2 \longrightarrow e^{-2\pi i L/M}\phi_2 \, , \tag{4.2}$$

where L is an odd integer and except for the cases $L = \pm 1$ the supersymmetry is completely broken. This orbifold was recently discussed in the context of closed string tachyon condensation [28]. Then we can speculate the same identification (3.12) in the untwisted sector. Here we should regard scalar fields Q^1 and Q^2 as $Q^1_{j,j+1}$, $(Q^2)_{j+L,j}$ instead of $(Q_1)_{j,j+1}(= Q^1_j)$, $(Q_2)_{j+1,j}(= Q^2_j)$, where we have written the indices of Chan-Paton matrices explicitly. The quantization of GS string on this non-supersymmetric background is also the same as in section 3.1 . In twisted sectors, however, the spectrum of string is slightly difficult to analyze because of the ambiguity of zero energy. In any case it will be intriguing fact that we can find string states in untwisted sector whose light-cone energy is independent of g. Thus there seem to exist the operators whose conformal dimensions are fixed as $\Delta - J = 0, 1, 2, \ldots$ in the large-N limit of the non-supersymmetric gauge theory if we believe the AdS/CFT correspondence in such a pp-wave limit.

5. Conclusions

In this paper we investigated an extension of the AdS/CFT correspondence in the limit of the orbifolded pp-wave background. We examined both the spectrum of states in the orbifolded Green-Schwarz string and the operators in the gauge theory side. The results show that the duality between string theory and $\mathcal{N} = 4$ super Yang-Mills theory discussed in [5] holds for less supersymmetric case ($\mathcal{N} = 2$ quiver gauge theory). We found an interesting interpretation of the twisted boundary condition in the orbifold theory from the viewpoint of the quiver gauge theory. We also discussed the similar issue in more general orbifolds and found that some parts of the results can be applied to non-supersymmetric orbifolds. It will be also interesting to analyze the pp-wave limit of D3-branes on the orbifolds C^3/Γ , which will lead to $\mathcal{N}=1$ quiver gauge theory (for some previous discussions see [22]).

Acknowledgments

The research of T.T was supported in part by JSPS Research Fellowships for Young Scientists.

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